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STABILITY OF A VISCOELASTIC PLATE IN FLUID FLOW

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The stability of an infinite viscoelastic plate on an elastic foundation in a viscous incompressible flow is studied. The Navier–Stokes system is linearized for an exponential velocity profile. The problem is reduced by a Fourier–Laplace transform to a system of ordinary differential equations, whose solution is found in the form of convergent series. The roots of the dispersion relation that characterize the stability of the system are found numerically. The effect of the viscosities of the fluid and the plate on the stability of the waves propagating upstream and downstream is studied. The results are compared with available data on the stability of a viscoelastic plate in an ideal fluid flow.

Key words: *viscoelastic plate, viscous fluid, exponential profile, stability of waves.*

Introduction. The problem of the interaction of a viscous fluid flow with the time-varying surface of an elastically deformable solid is important from both practical and theoretical points of view. Despite many papers on this problem, many issues related to the stability of the examined mechanical system have not been adequately studied. Experimental verification of the theoretical results indicating intense vibrations of structural members in flow of a viscous continuous medium (gas or liquid) is extremely difficult. In each particular case, a special mathematical model is constructed and the type of instability depends appreciably on the choice of a physical model (see, for example, [1–5]). In the present paper, we study in detail the effect of internal friction (structural damping) and external friction (fluid viscosity) on the stability of a viscoelastic plate on an elastic foundation. Generally, the chosen physical model is characterized by two types of instability: divergence and panel flutter. These phenomena have been well studied for a plate in an ideal incompressible fluid flow with a velocity profile constant in both time and coordinates. In the present work, the description of these phenomena is refined by a detailed consideration of the waves propagating both downstream and upstream. Accounting for external (or internal) friction in the nonconservative problem in question leads to rather ambiguous results (see, for example, [6]). Our studies have shown that these two types of friction differently influence the stability of the waves propagating in opposite directions.

For the viscous fluid model, the main velocity profile was assumed to be exponential. As is known, this profile can be produced asymptotically by suction of fluid from the boundary-layer flow over a motionless surface [7]. Theoretical and experimental studies have shown that the exponential structure is stable. At the same time, real profiles can be well approximated by a linear combination of exponential profiles with appropriate parameters.

In this paper, we propose a convenient method for the numerical solution of the problem for the case of exponential-velocity flow over the vibrating surface of a plate. It is shown that the parameters of the elastic body have a significant influence on the stability of the system. In addition, the explicit solution of the equation for an ideal fluid and its comparison with the results obtained numerically for a viscous fluid suggest that there is a fundamental difference in the behavior of the waves propagating (downstream and upstream) in viscous and ideal fluid flows over a plate.

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1. Formulation of the Problem. We assume that the flow depends only on the vertical coordinate; therefore, the problem can be treated as the problem of two-dimensional flow above an infinite vibrating beam (Bernoulli–Euler beam) on an elastic foundation.

Generally, the problem of the flow $\boldsymbol{u}^*$ over an elastic beam in a half-plane is formulated as follows. It is required to find the shape of the beam $\Gamma(w(y))$, the velocity field $\boldsymbol{V} = (V_1, V_2)$, and the function of the fluid pressure p that satisfy the Navier–Stokes equations

$$\begin{gathered} \dot{\boldsymbol{V}} + (\boldsymbol{V} \cdot \nabla)\boldsymbol{V} - \nu\Delta\boldsymbol{V} + \rho^{-1}\nabla p = 0, \\ \nabla \cdot \boldsymbol{V} = 0 \qquad \text{in} \quad \Omega = \{(x, y)\colon y > w\}, \end{gathered} \tag{1.1}$$

and the initial and boundary conditions

$$\boldsymbol{V}\Big|_{\Gamma} = \boldsymbol{v}_b, \qquad \lim_{y\to+\infty} \boldsymbol{V} = \boldsymbol{u}_0, \qquad \boldsymbol{V}\Big|_{t=0} = \boldsymbol{V}_0. \tag{1.2}$$

In (1.1) and (1.2), $\nabla = (\partial/\partial x, \partial/\partial y)$, $\Delta = \nabla \cdot \nabla$, the positive constants ν are ρ are the kinematic viscosity of the fluid and its density, respectively, $\boldsymbol{v}_b$ is the velocity of the beam, $\boldsymbol{u}_0 = (u_0, 0)$ is the limiting flow velocity at infinity, $\boldsymbol{V}_0$ is the initial velocity field (the dot above the symbol denotes the time derivative), and w is the deviation of the beam from the zero position $y = 0$ (it is assumed that the beam can have only a vertical deviation).

In addition, we assume that the deviation w satisfies the well-known equation

$$m\ddot{w} + D(w'''' + \varkappa\dot{w}'''') + Kw = \boldsymbol{e}_y \cdot T\boldsymbol{n} \qquad (t > 0), \tag{1.3}$$

where the prime denotes the derivative with respect to the horizontal coordinate x, m and D are the mass per unit length of the beam and it flexural rigidity, respectively, K is the rigidity coefficient of the elastic foundation, $\varkappa$ is a coefficient that characterizes the internal friction of the beam, $\boldsymbol{n}$ is the normal to Ω that is external with respect to the region Γ, $\boldsymbol{e}_y$ is the unit vector on the y axis, and T is the stress tensor with the components

$$T_{ik}(\boldsymbol{V}, p) = -p\delta_i^k + \mu\Big(\frac{\partial V_i}{\partial x_k} + \frac{\partial V_k}{\partial x_i}\Big) \qquad (i = 1, 2; \quad k = 1, 2).$$

Here δ_i^k is the Kronecker symbol, $x_1 \equiv x$, $x_2 \equiv y$, and $\mu = \nu\rho$ is the dynamic viscosity of the fluid. As $x \to \infty$, the function w and its derivatives are considered bounded.

2. Linearization of System (1.1)–(1.3) for Flow with an Exponential Velocity Profile. We linearize system (1.1)–(1.3) for the vector $\boldsymbol{u}^* = (u^*(y), 0)$, where $u^*(y) = -u_0(1 - \mathrm{e}^{-\lambda y})$ $(\lambda > 0)$, and write the boundary-value problem for the perturbations $(\boldsymbol{u}, q, w)$ in the half-plane $\mathbb{R}^2_+$ $(y > 0)$:

$$\begin{gathered} \dot{\boldsymbol{u}} - \nu\Delta\boldsymbol{u} + (\boldsymbol{u}^* \cdot \nabla)\boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}^* + \rho^{-1}\nabla q = 0, \\ \nabla \cdot \boldsymbol{u} = 0 \qquad \text{in} \quad \mathbb{R}^2_+ \times (0, \infty); \end{gathered} \tag{2.1}$$

$$u_1\Big|_{y=0} = 0, \qquad u_2\Big|_{y=0} = \dot{w}, \qquad \lim_{\substack{y\to+\infty \\ |x|\to\infty}} \boldsymbol{u} = 0; \tag{2.2}$$

$$m\ddot{w} + D(w'''' + \varkappa\dot{w}'''') + Kw - \mu u_0\lambda w' = -q\Big|_{y=0}, \qquad \lim_{|x|\to\infty} w = 0. \tag{2.3}$$

Let $\boldsymbol{u}_0 = (-u_0, 0)$ be a constant vector $(u_0 \geq 0)$. Equation (2.1) can be written as

$$\begin{gathered} \dot{\boldsymbol{u}} - \nu\Delta\boldsymbol{u} + (\boldsymbol{u}_0 \cdot \nabla)\boldsymbol{u} + \rho^{-1}\nabla q = ((\boldsymbol{u}_0 - \boldsymbol{u}^*) \cdot \nabla)\boldsymbol{u} - (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}^*, \\ \nabla \cdot \boldsymbol{u} = 0 \qquad \text{in} \quad \mathbb{R}^2_+ \times (0, \infty). \end{gathered} \tag{2.4}$$

We assume that the problem (2.4), (2.2), (2.3) admits a solution $(\boldsymbol{u}, q, w)$ with an exponential time dependence $\boldsymbol{u}(x, y, t) = \hat{\boldsymbol{u}}(x, y, s)\,\mathrm{e}^{st}$, $q(x, y, t) = \hat{q}(x, y, s)\,\mathrm{e}^{st}$, and $w(x, t) = \hat{w}(x, s)\,\mathrm{e}^{st}$. To determine at what s this regime is possible, we substitute this solution into (2.4), (2.2), and (2.3) and apply a Fourier transform to the variable x using the formula

$$\tilde{f}(\xi) = \int\limits_{-\infty}^{+\infty} f(x)\,\mathrm{e}^{-i\xi x}\,dx. \tag{2.5}$$

As a result, we obtain the following boundary-value problem for the system of ordinary differential equations (for the variable y) for the images of the velocity field $\tilde{\boldsymbol{u}}(\xi, y, s)$, pressure $\tilde{q}(\xi, y, s)$, and beam displacements $\tilde{w}(\xi, s)$:

$$\frac{d^2\tilde{u}_1}{dy^2} - r_1^2\tilde{u}_1 - \frac{i\xi\tilde{q}}{\mu} = \frac{i\xi u_0}{\nu}\, \mathrm{e}^{-\lambda y}\, \tilde{u}_1 - \frac{u_0\lambda}{\nu}\, \mathrm{e}^{-\lambda y}\, \tilde{u}_2,$$

$$\frac{d^2\tilde{u}_2}{dy^2} - r_1^2\tilde{u}_2 - \frac{1}{\mu}\frac{d\tilde{q}}{dy} = \frac{i\xi u_0}{\nu}\, \mathrm{e}^{-\lambda y}\, \tilde{u}_2, \tag{2.6}$$

$$i\xi\tilde{u}_1 + \frac{d\tilde{u}_2}{dy} = 0;$$

$$\tilde{u}_1(\xi, 0, s) = 0, \qquad \tilde{u}_2(\xi, 0, s) = s\tilde{w}, \qquad \lim_{y\to+\infty} \tilde{\boldsymbol{u}} = 0; \tag{2.7}$$

$$\Big(ms^2 + (1 + \varkappa s)D\xi^4 + K - i\mu u_0\lambda\xi\Big)\tilde{w} = -\tilde{q}(\xi, 0, s). \tag{2.8}$$

Here $r_1 = \sqrt{(s - i\xi u_0)/\nu + \xi^2}$ and it is assumed that $\mathrm{Re}\sqrt{z} \geq 0\ \forall\ z \in \mathbb{C}$.

As shown in [8], a solution of (2.6), (2.7) can be found in the form of exponential series

$$\tilde{u}_1(\xi, y, s) = -\frac{B_1(0)s\tilde{w}}{P}\, \mathrm{e}^{-r_1 y}\, A_1(y) + \frac{A_1(0)s\tilde{w}}{P}\, \mathrm{e}^{-|\xi|y}\, B_1(y),$$

$$\tilde{u}_2(\xi, y, s) = -\frac{B_1(0)s\tilde{w}}{P}\, \mathrm{e}^{-r_1 y}\, A_2(y) + \frac{A_1(0)s\tilde{w}}{P}\, \mathrm{e}^{-|\xi|y}\, B_2(y), \tag{2.9}$$

$$\tilde{q}(\xi, y, s) = -\frac{B_1(0)s\tilde{w}}{P}\, \mathrm{e}^{-r_1 y}\, A_3(y) + \frac{A_1(0)s\tilde{w}}{P}\, \mathrm{e}^{-|\xi|y}(B_3(y) + B_4),$$

where

$$P = A_1(0)B_2(0) - B_1(0)A_2(0),$$

$$A_1(y) = \sum_{k=0}^{\infty} \mathrm{e}^{-k\lambda y}\, a_1^{(k)}, \quad A_2(y) = \sum_{k=0}^{\infty} \mathrm{e}^{-k\lambda y}\, a_2^{(k)}, \quad A_3(y) = \sum_{k=0}^{\infty} \mathrm{e}^{-k\lambda y}\, a_3^{(k)},$$

$$B_1(y) = \sum_{k=0}^{\infty} \mathrm{e}^{-k\lambda y}\, b_1^{(k)}, \quad B_2(y) = \sum_{k=0}^{\infty} \mathrm{e}^{-k\lambda y}\, b_2^{(k)}, \quad B_3(y) = \sum_{k=0}^{\infty} \mathrm{e}^{-k\lambda y}\, b_3^{(k)}, \quad B_4 = \frac{\rho(s - i\xi u_0)}{i\xi};$$

$$\begin{gathered}
a_1^{(k)} = \Big(\frac{i\xi u_0}{\nu\lambda}\Big)^k \frac{r_1 + k\lambda}{k!r_1}\, R_k, \qquad a_2^{(k)} = \Big(\frac{i\xi u_0}{\nu\lambda}\Big)^k \frac{i\xi}{k!r_1}\, R_k, \\
a_3^{(k)} = \Big(\frac{i\xi u_0}{\nu\lambda}\Big)^k \frac{2\lambda^2\mu i\xi}{(k-1)!r_1((r_1 + k\lambda)^2 - \xi^2)}\, R_{k-1}, \\
b_1^{(k)} = \Big(\frac{i\xi u_0}{\nu}\Big)^k \frac{|\xi| + k\lambda}{k!|\xi|}\, Q_k, \qquad b_2^{(k)} = \Big(\frac{i\xi u_0}{\nu}\Big)^k \frac{i\xi}{k!|\xi|}\, Q_k, \\
b_3^{(k)} = \Big(\frac{i\xi u_0}{\nu}\Big)^k \frac{2\mu i\xi}{k!|\xi|(2|\xi| + k\lambda)}\, Q_{k-1};
\end{gathered} \tag{2.10}$$

$$R_k = \prod_{j=1}^{k} \frac{(r_1 + j\lambda)(r_1 + (j-2)\lambda) - \xi^2}{(2r_1 + j\lambda)((r_1 + j\lambda)^2 - \xi^2)}, \qquad R_0 \equiv 1,$$

$$Q_k = \prod_{j=1}^{k} \frac{2(j-1)|\xi| + j(j-2)\lambda}{(2|\xi| + j\lambda)((|\xi| + j\lambda)^2 - r_1^2)}, \qquad Q_0 \equiv 1.$$

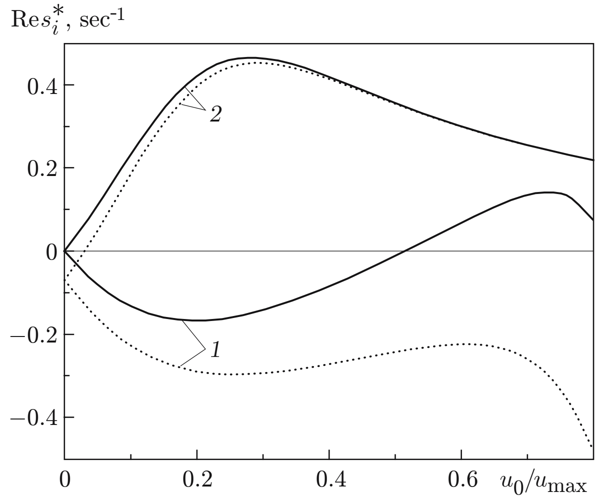

Fig. 1

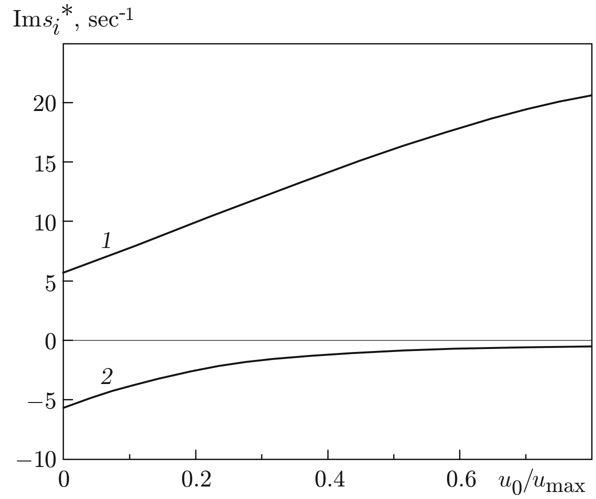

Fig. 2

Fig. 1. Real parts of the roots s_i^* of the dispersion equation (2.12) for the case of a viscous fluid: the solid and dotted curves refer to $\varkappa = 0$ and 0.01 sec; curves 1 refer to the wave moving downstream and curves 2 refer to the wave moving upstream.

Fig. 2. Imaginary parts of the roots s_i^* of the dispersion equation (2.12) for the case of a viscous fluid (notation the same as in Fig. 1).

In the expressions for R_k and Q_k, the powers of the numerators are smaller than those of the denominators, and, hence, their moduli do not exceed const$/k!$. Therefore, the moduli $|a_i^{(k)}|$ and $|b_i^{(k)}|$ in (2.10) are bounded from above by the product of the constant and the general term of the Taylor series for the exponential function. Thus, it can be concluded that the series in (2.9) converge for all ξ and u_0, and they converge uniformly for y because $\mathrm{e}^{-k\lambda y} \le 1$ and $y \ge 0$.

Substitution of $\tilde{q}(\xi, 0, s)$ into the beam elasticity equation (2.8) yields the following condition for the existence of a nontrivial solution $\tilde{w}$ for system (2.6)–(2.8):

$$ms^2 + K + (1 + \varkappa s)D\xi^4 - i\mu u_0\lambda\xi + \frac{-A_3(0)B_1(0) + (B_3(0) + B_4)A_1(0)}{P}\, s = 0. \qquad (2.11)$$

Multiplying the series, we obtain the determinant P in the form

$$P = \sum_{n=0}^{\infty} \Big(\frac{i\xi u_0}{\nu\lambda}\Big)^n \frac{i\xi}{r_1|\xi|} \sum_{k=0}^{n} \frac{\lambda^{n-k}(r_1 - |\xi| + (2k-n)\lambda)}{k!(n-k)!}\, R_k Q_{n-k}.$$

Multiplying Eq. (2.11) by $i\xi P$ and again multiplying the series by each other, we obtain the dispersion relation

$$(ms^2 + K + (1 + \varkappa s)D\xi^4 - i\mu u_0\lambda\xi)i\xi P + \frac{\rho s(s - i\xi u_0)r_1}{|\xi|} + \sum_{n=1}^{\infty}\Big(\frac{i\xi u_0}{\nu\lambda}\Big)^n \Big\{\frac{\rho s(s - i\xi u_0)(r_1 + n\lambda)}{|\xi| n!}\, R_n$$

$$+ 2\mu s \sum_{k=0}^{n-1} \frac{\lambda^{n-k}}{k!(n-k-1)!}\Big(\frac{r_1 + k\lambda}{(n-k)(2|\xi| + (n-k)\lambda)} - \frac{\lambda(|\xi| + (n-k-1)\lambda)}{(r_1 + (k+1)\lambda)^2 - \xi^2}\Big) R_k Q_{n-k-1}\Big\} = 0. \qquad (2.12)$$

The roots s_i^* of the dispersion equation (2.12) indicate an exponential time regime, whose existence was suggested above. The stability of the fluid–beam system is defined by the signs of the real parts of the roots s_i^*.

3. Stability Analysis. Numerical studies were performed for a particular model with parameters $m = 80$ kg/m^2, $D = 15{,}000$ N$\cdot$m, $K = 20{,}000$ N/m^3, $\nu = 10^{-6}$ m^2/sec, $\rho = 1000$ kg/m^3, $\lambda = 1000$ m^{-1}, and $\xi = 1$ m^{-1}. The velocity range $u_0 = 0$–25 m/sec was chosen.

Figure 1 shows a curve of the real parts of the roots s_i^* ($i = 1, 2$) of the dispersion equation (2.12) versus the flow velocity u_0 referred to $u_{\max} = 25$ m/sec for zero (solid curve) and nonzero (dotted curve) internal viscosity of the beam. Figure 2 gives similar dependences for the imaginary parts of the roots s_i^*. It is obvious that for

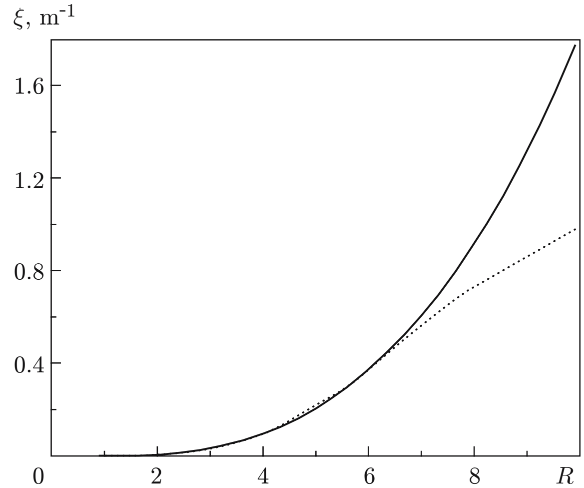

Fig. 3. Neutral stability curve for Eq. (2.12) (dotted curve) and its asymptotic approximation (solid curve).

$u_0 = 0$, Eq. (2.12) has two complex conjugate roots with a negative real part, which corresponds to the presence of two damping waves in the fluid that propagate in the opposite directions: downstream (the root s_1^* with a positive imaginary part) and upstream (the root s_2^* with a negative imaginary part). As the velocity u_0 increases for $\varkappa = 0$, the quantity $\operatorname{Re} s_2^*$ is the first to become positive, after which the quantity $\operatorname{Re} s_1^*$ becomes positive, i.e., the wave propagating upstream is the first to become unstable. In the presence of internal viscosity, the wave propagating upstream becomes unstable for a larger value of the velocity. The wave propagating downstream remains stable for all values of u_0. The internal viscosity has little effect on the imaginary parts of the roots or on the phase velocities of the waves (the solid and dotted curves in Fig. 2 coincide).

Thus, in the presence of fluid viscosity ν, flutter instability of the system occurs, and the presence of internal viscosity $\varkappa$ increases the stability of both waves.

We study the behavior of the roots of the dispersion equation (2.12) for various values of ξ. For small ξ, since the roots s_i^* are difficult to calculate, we consider the asymptotic approximation of relation (2.12) for $\xi \to 0$ with accuracy up to $O(\xi)$:

$$\Delta(s) \equiv \frac{\rho s^2}{|\xi|} + K + ms^2 + \rho s\nu\sqrt{\frac{s}{\nu}} - \frac{i\rho s u_0 \lambda}{\lambda + \sqrt{s/\nu}} = 0. \tag{3.1}$$

Let us elucidate when the equation $\Delta(s) = 0$ can have purely imaginary roots. For this, we substitute into it $s = \pm i\omega$, where $\omega > 0$, and divide the real and imaginary parts of the last expression:

$$\operatorname{Re}\Delta(\pm i\omega) \equiv -\frac{\rho\omega^2}{|\xi|} + K - m\omega^2 - \rho\omega\nu\sqrt{\frac{\omega}{2\nu}} \pm \frac{\rho\omega u_0\lambda(\lambda + \sqrt{\omega/(2\nu)}\,)}{\lambda^2 + \lambda\sqrt{2\omega/\nu} + \omega/\nu} = 0,$$

$$\operatorname{Im}\Delta(\pm i\omega) \equiv \pm\rho\omega\nu\sqrt{\frac{\omega}{2\nu}} - \frac{\rho\omega u_0\lambda\sqrt{\omega/(2\nu)}}{\lambda^2 + \lambda\sqrt{2\omega/\nu} + \omega/\nu} = 0.$$

From the equation for $\operatorname{Im}\Delta(\pm i\omega)$, we obtain

$$u_0 = \pm\Big(\lambda\nu + \sqrt{2\omega\nu} + \omega/\lambda\Big). \tag{3.2}$$

Substitution of (3.2) into the condition $\operatorname{Re}\Delta(\pm i\omega) = 0$ yields the relation

$$|\xi| = \rho\omega^2/(K - m\omega^2 + \rho\omega\nu\lambda). \tag{3.3}$$

Thus, $|\xi(\omega)|$ does not depend on the choice of the sign of u_0 and quantity $-\omega$ corresponds to negative values of the velocity u_0. Since $u_0 \geq 0$, from (3.2) we find

$$\omega = \lambda u_0 - \lambda\sqrt{\lambda\nu(2u_0 - \lambda\nu)}. \tag{3.4}$$

Because relation (3.2) implies that $u_0 > \nu\lambda$, the radicand in (3.4) is always positive. Hence, expression (3.4) defines the real values $\omega > 0$ and the dependence between $|\xi|$ and u_0, defined by relations (3.3) and (3.4), specifies the neutral stability curve for Eq. (3.1), which is shown by the solid curve in Fig. 3. The dotted curve corresponds to the neutral curve obtained by numerical solution of Eq. (2.12) [with the Reynolds's number $R = u_0/(\lambda\nu)$ used as the parameter]. We note that the region of instability is below these curves. For small ξ, the curves almost coincide, i.e., the values of the purely imaginary root of the approximation (3.1) coincide with the values of the root s_2^* found numerically for the corresponding values of the parameters ξ and R. For small values of R, a second purely imaginary root s_1^* does not exist; neither does a purely imaginary root s_2^* exist for $R < 1$, i.e., in this case, the fluid–beam system is stable for any values of ξ.

To determine the effect of the fluid viscosity ν on the stability of the system, we compare the obtained results with similar results for the case of an inviscid fluid. Ideal fluid flow over a flexible beam has been thoroughly studied (see, for example, [1, 6]).

The perturbation potential φ of the ideal-fluid velocity $\boldsymbol{u} = \nabla\varphi$ is a solution of the problem

$$\Delta\varphi = 0, \qquad \lim_{y\to\infty} \varphi = O(1), \qquad \frac{\partial\varphi}{\partial y}\Big|_{y=0} = \dot{w} - u_0 w'; \tag{3.5}$$

$$m\ddot{w} + D(w'''' + \varkappa\dot{w}'''') + Kw = -\rho(\dot{\varphi} - u_0\varphi')\Big|_{y=0}. \tag{3.6}$$

Assuming an exponential time regime and performing a Fourier transform (2.5) in system (3.5), (3.6), we obtain the image of the potential φ:

$$\tilde{\varphi}(\xi, y, s) = -\frac{s - i\xi u_0}{|\xi|}\,\tilde{w}\,\mathrm{e}^{-|\xi| y}.$$

Substitution of this solution into the image of condition (3.6) yields the dispersion relation for an inviscid fluid:

$$m|\xi|s^2 + D|\xi|^5(1 + \varkappa s) + K|\xi| + \rho(s - i\xi u_0)^2 = 0. \tag{3.7}$$

We first consider the case $\varkappa = 0$ (the absence of viscosity of the beam). Solving the quadratic equation (3.7) for s, we obtain the roots

$$s_{1,2} = \frac{i\xi u_0\rho \pm \sqrt{m|\xi|^3\rho u_0^2 - (m|\xi| + \rho)(D|\xi|^5 + K|\xi|)}}{m|\xi| + \rho}.$$

For $u_0 \le u_f = \sqrt{(D|\xi|^4 + K)/(m|\xi|^2) + (D|\xi|^4 + K)/(\rho|\xi|)}$, both roots are purely imaginary, and, in addition, for $u_0 = u_f$, they are equal. For $u_0 > u_f$, the real part of s_1 becomes positive, s_2 becomes negative, and flutter instability of the system occurs. Curves of the real and imaginary parts of the roots of Eq. (3.7) versus the flow velocity u_0 in the absence of internal friction $\varkappa$ are given in Fig. 4a and Fig. 4b, respectively (solid curves). Thus, in the absence of both viscosities, for any value of the wave number $\xi \ne 0$ there is a range of velocity values in which two waves with neutral stability exist in the fluid.

If the viscosity of the beam $\varkappa > 0$, Eq. (3.7) has the roots

$$s_{1,2} = \frac{2i\xi\rho u_0 - \varkappa D|\xi|^5}{2(m|\xi| + \rho)} \pm \frac{\sqrt{(\varkappa D|\xi|^5 - 2i\xi\rho u_0)^2 - (m|\xi| + \rho)(D|\xi|^5 + K|\xi| - \rho\xi^2 u_0^2)}}{2(m|\xi| + \rho)}. \tag{3.8}$$

From Eq. (3.8) it follows that in the case of small $\varkappa$ (which corresponds to the real values of the internal viscosity) and $u_0 = 0$, the dispersion equation has two complex conjugate roots with a negative real part. This suggests that in the fluid there are two damping waves propagating in opposite directions. In this sense, the nature of the waves in stagnant water is similar to the nature of the waves for the case of a viscous fluid. As the flow velocity u_0 increases, the real part of the root with a negative imaginary part s_2 increases, and for $u_0 = u_d \equiv \sqrt{(D|\xi|^4 + K)/(\rho|\xi|)}$, it becomes equal to zero. In this case, the imaginary part s_2 also becomes equal to zero. With a further increase in u_0, both $\operatorname{Re} s_2$ and $\operatorname{Im} s_2$ become positive. Hence, the wave propagating downstream experiences divergent instability at the point $u_0 = u_d$. The wave propagating upstream remains stable for all values of u_0. Plots of the real and imaginary parts of the roots s_1 and s_2 for $\varkappa = 0.01$ sec are given in Fig. 4 (dotted curves).

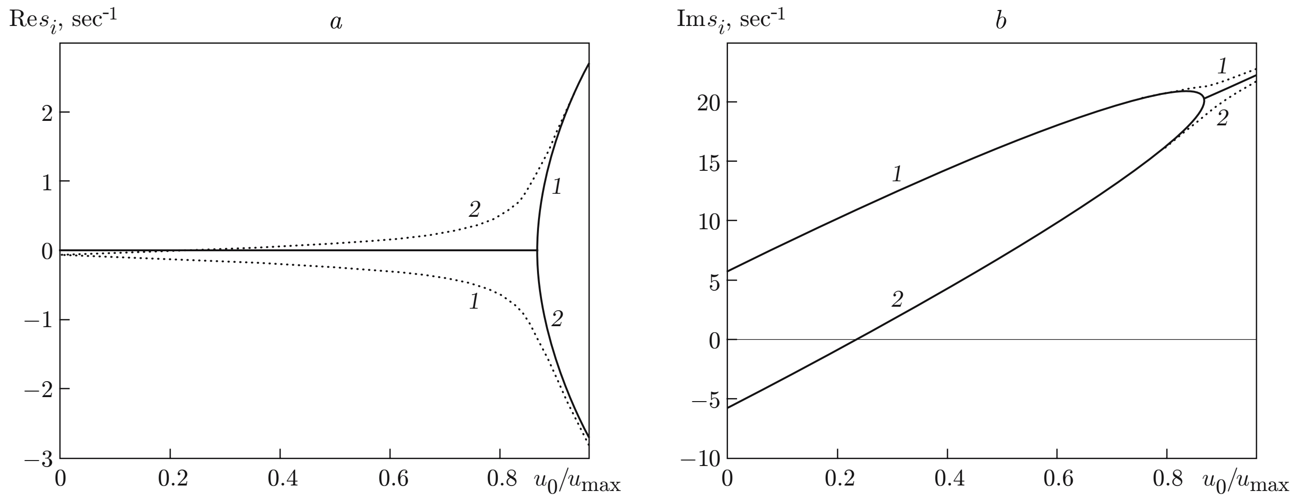

Fig. 4. Real (a) and imaginary (b) parts of the roots s_i of the dispersion equation (3.7) for the case of an inviscid fluid: curves 1 refer to the downstream propagating wave and curves 2 refer to the upstream propagating wave; the solid and dotted curves refer to $\varkappa = 0$ and 0.01 sec, respectively.

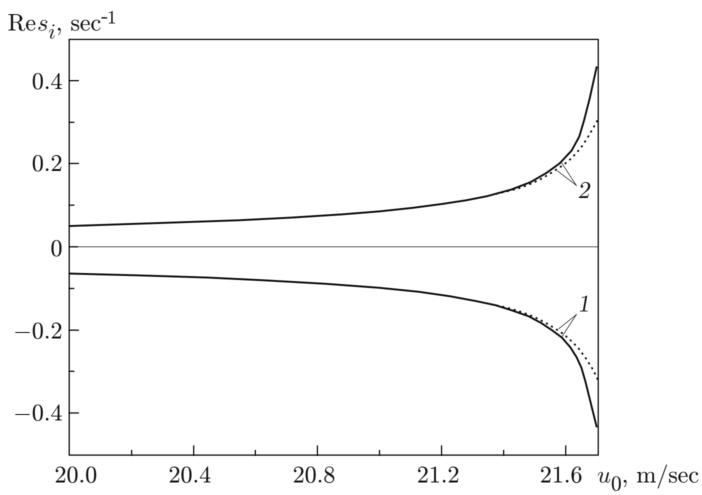

Fig. 5. Real parts of roots (3.8) (dotted curves) and their asymptotic approximations (3.9) (solid curves): curves 1 and 2 refer to the downstream and upstream propagating wave, respectively.

To describe the damping effect, we give an asymptotic expansion of the real part of expression (3.8) in powers of $\varkappa$:

$$\mathrm{Re}\,(s_{1,2}) \approx -\frac{\varkappa D|\xi|^5}{2(m|\xi| + \rho)}\Big(1 \pm \frac{1}{\sqrt{A}}\,|\xi|\rho u_0\Big) \qquad \text{at} \quad u_0 < u_f. \tag{3.9}$$

Here $A = (m|\xi| + \rho)(D|\xi|^5 + K|\xi|) - m|\xi|^3\rho u_0^2$. For the examined values of $\varkappa$, this expansion almost coincides with the real part of expression (3.8), except in the vicinity of the point $u_0 = u_f$, where the expression of A vanishes (Fig. 5).

From (3.9) it follows that at small velocities u_0, the quantities $\mathrm{Re}\,(s_{1,2})$ depend linearly on $\varkappa$. The product $\varkappa u_0$ indicates stabilization of the downstream wave with increasing $\varkappa$, although the velocity $u_0 = u_d$ at which the wave propagating upstream and, hence, the entire system become unstable does not depend on $\varkappa$. In addition, u_d is always smaller than u_f, and, therefore, in the presence of internal viscosity, the system generally becomes unstable sooner than it does without viscosity.

The results of the studies performed suggest that for the specified parameters of the beam in the case of viscous and inviscid fluids, the internal viscosity $\varkappa$ stabilizes the wave propagating downstream. As regards the wave propagating upstream, its stabilization is observed only at small velocities. The external viscosity ν destabilizes the wave propagating upstream: it rapidly loses stability; at the same time, the neutral wave propagating downstream becomes damping but then, as the flow velocity u_0 increases, it nevertheless loses stability. Thus, the effect of the viscous terms on the vibration waves leads to a significant change in the flow pattern.

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